HEIGHT REFERENCE OF TERRAIN RELIEFS

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Presently the geo-referenced digital *terrain relief* received a crucial and duplicated role as a geographical variable and as parameter for objectives, related to dynamical processes within the Earth's system. The accuracy and resolution of topographic heights in the terrain reliefs has a special importance in meteorology, hydrology and other Earth's disciplines dealing with dynamical processes. The topographic height referencing comes from several sources starting from terrestrial leveling and map digitalization till different remote measuring techniques. The accuracy with respect to a *common* geodetic reference coordinate frame depends on several factors such as measuring instrumentation, vegetation covering and slope of the land, local gravity anomalies and the homogeneity of applied geodetic datum on the covered area

The locations of weak accuracy on a territory can be revealed by comparing the leveled topographic relief to those of others produced by some different method. Recently the topographic reliefs are frequently produced from GPS data and this idea can be applied for their qualification. Two territories were selected (one in Hungary and one in Greece) for demonstration where both GPS and leveled data were available. The GPS topographic heights were calculated by using the spherical harmonic model with ellipsoid WGS84 and gravity field model EIGEN-GL04C. Reliefs of difference between GPS and leveled topographic heights have indicated local gravity anomalies where the accuracy of terrain reliefs produced from GPS data are weak. The shape of relief of height difference is similar to that of geoid height on Greek territory that indicates a height datum problem.

Keywords: terrain relief, height referencing, ellipsoidal height, topographic height, geoid undulation, Earth's gravity field model, spherical harmonic model, local gravity anomalies, geodetic datum

1. Introduction

Geo-referencing "objects" or "information" is a basic need for all the Earth and Engineering disciplines including environmental perspectives of the Earth's use. The exact position of "something" within the physical space in analogue or digital form can be crucial for particular applications. This demand is important for global, regional and local scale objectives that deal with the monitoring of dynamical processes within the Earth's system. For such applications the geo-referencing has to be based on reference coordinate frames expressed with respect to a particular Earth's datum.

Earth disciplines such as geodesy and surveying have been configured in the past to provide high global *positional* accuracy and local high-resolution for digital *terrain or topographic relief*. This last objective has been greatly assisted by satellite imaging techniques in any geographical scale during the past 40 years. Presently the terrain relief received a crucial and duplicated role as a geographical variable and as parameter for objectives, related to dynamical processes within the Earth's system. Under this role the terrain relief is subjected to various demands for accurate and/or well resolved representation forms in point or pixel forms. Thus, nowadays the approach to represent the terrain relief in form of topographic mapping is configured by many new methods based on space science. The coming out geo-referenced "products" (e.g. global geoid, satellite imaging, Digital Earth Models) are then principal sources that contribute to digital maps of which the accuracy is evaluated at particular reference point coordinates. Out of these hybrid 3D reference coordinate frames, the height coordinate is the most vulnerable to maintain a level of quality of geo-referencing on the regional or local scale because the reference heights are associated with the quality and the type of the used height reference datum. The used height reference datum can be a mathematical or a physical Earth's model, which are interrelated in many forms.

Therefore, in spite of the presently available alternative technologies of geo-referencing, there exist many source data that have an impact on the overall positional accuracy, mainly to *topographic heights,* which can be crucial in developing forms of digital topographic data base for GIS. The reason is that the topographic maps still count as the main data source for the adaptation and the evaluation of the performance of aerial or satellite imaging methods. The scale of map and the geographical rendering area representation type are quite important aspects because the *information content* about the *terrain relief* depends mainly on the scale set and the resulting *local dependence of map's representations.* Topographic heights H are measured with respect to a physical Earth's model, which expresses a conventional Mean Sea Level, based on various determinations of it in connection to a particular Earth's ellipsoid.

Thus, the *uncertainty sources of the topographic heights* can be a significant problem in a GIS *because particular spatial data sets* as pollution measures, weather indices (air direction, barometric pressure, humidity), mass transfer issues (air, water, oil spots, landslides), are used for purposes for which they have been intended never before. From this perspective, the dual importance of height referencing as height coordinate and as geographical variable/parameter brings up questions about the real accuracy and resolving capacity of a particular height data set, which are complementary measures of the terrain relief forms.

Map accuracy has been of major importance in geodesy and other Earth's mapping disciplines, which focus on accuracy issues related to the analog mapping at reference points for the 3 coordinates. When the same map is digitized and used as input into GIS, *the mode of the map's use often changes* and its new uses extend beyond a determined domain for which the original map was intended. Most studies about accuracy of terrain relief are specialized for the accuracy that can be achieved by radar-based acquiring data methods compared to "bald" Digital Elevation Models. An evaluation of this accuracy is not relevant to the accuracy that characterizes the *accuracy of the basic reference height systems*, which remains a subject of geodetic interest.

The accuracy of terrain relief when it is considered as an *environmental variable/parameter* can be evaluated from existing maps or by geodetic reference coordinate frames in association with a particular measuring instrumentation. For instance, for the portable terrestrial interferometers a recent study has shown that at distance 2 km from the radar sensor, *the height error standard deviation is about 3* m. (Strozzi et al. 2012). Other types of errors can be higher for the remote sensed areas, where the type of vegetation, the slope of radar beam or the rate of the land elevation affect the height accuracy in various ways (Rees 2000). Another type of error in heights may be present in data that come from the merging of different height data sources in a common file and in particular when the sources are based on different frames of geo-referencing. For regional monitoring it is essential to express all the data sets in a common referencing frame and this objective becomes crucial for the height coordinate. A principal cause comes from the conversion of the two types of heights; namely the *ellipsoid height* h and the *topographic height* H and in particular when different height data sets are referenced to different Earth's models.

The interrelation of the two types of height by the simplified relation:

$$h(\varphi,\lambda) = N(\varphi,\lambda)+H(\varphi,\lambda) \qquad (1)$$

where $N(\varphi,\lambda)$ expresses a height between the Earth's zero level physical datum (geoid or Mean Sea Level) and a corresponding mathematical datum (ellipsoid). It may be confusing because the used values for $N(\varphi,\lambda)$ can refer to different Earth's height datum than the one for the physical height, H. It can cause further confusion that for the same datum the accuracy to determine the height $N(\varphi,\lambda)$, depends on the method, on data sets and on the real impact of the local gravity field. This accuracy may vary from a few cm to several meters. Any combination of topographic heights H in a larger region determined by different methods can have different quality in accuracy and resolving capacity with respect to a common reference height system. Any reference height system reveals the impact of physical and structural factors that relate to the Earth.

The establishment of a common referencing frame for topographic heights H is not a simple process. There are difficulties arising: a) from the parallel use of both types of height systems b) from various accuracies, the resolving capacity and forms of the illustrated topographic heights from different data sets c) when the demands for accuracy and resolving capacity of a common height referencing frame associate with the terrain relief from the perspective of a geographical variable/parameter. In fact, the illustration of a topographic relief is the most vulnerable geographical "information" for a local and regional scale.

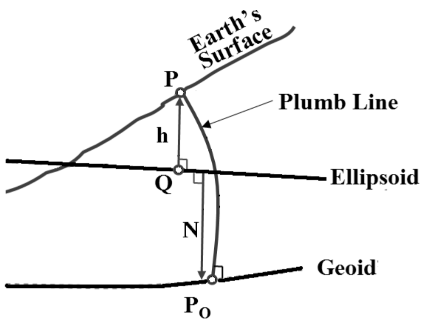

Fig.1 Ellipsoid height h, geoid height N, topographic height H=h-N

The representation of a *terrain relief* for geo-referencing in a territory subjected to dynamical processes should be based on the uniform accuracy of a height reference datum all over the territory. The accuracy of representation will depend on how well the local height reference matches a global height datum. Measures of this matching can be deduced from the combinations of values of heights $N(\varphi,\lambda)$ with respect to a local and a global height reference.

The paper is based on the use of both types of heights h and H with the aim to benefit from their interrelation, which illustrates *a form of dualism between the mathematical and the physical Earth's models.*

2. Two types of height systems

For general users or experts of GIS, conceptual or formal differences of geographical $h(\varphi, \lambda)$ and physical heights $H(\varphi,\lambda)$ are usually interpreted as a simple computation of $N(\varphi,\lambda)$ by Eq. (1) at all GPS observed positions. This computation is part of the processing algorithms of the GPS receivers. It is based on using sets of coefficients C_{ij}, S_{ij} that form a particular global gravity field model in a spherical expansion of functions that are rigorously related to the geo-potential of the Earth.

The computation of height $N(\varphi,\lambda)$ of Mean Sea Level (MSL) with respect to an Earth's ellipsoid datum is based on the next formula (Hofmann-Wellenhof and Moritz 2005) where the coefficients are calculated from satellite measurements (Barthelmes 2009a)

$$N(\varphi, \lambda)_{computed} = [kM/R\gamma] \, [\sum_{i=2}^{n} (a/R)^{i} \sum_{j=0}^{n} [C_{ij}\cos j\lambda + S_{ij}\sin j\lambda] \, P_{ij}(\sin\varphi) \qquad (2)$$

For GPS receivers the processing of an ellipsoid height $h(\varphi,\lambda)$ for a particular datum (e.g. the WGS84 or any other) into topographic H with respect to an associated *global representation* of MSL provides heights $H(\varphi,\lambda)_{computed}$ which have a nominal resolving capacity in a ground pixel $d\varphi d\lambda$ of several decades of km. This resolution depends on the degree n of the model's expansion. Thus, such converted height $H(\varphi,\lambda)_{computed}$ differs from a directly surveyed or determined height $H(\varphi,\lambda)_{measured}$ at the same position φ,λ by any other method. The difference δH between the two values is caused by physical, computational and technology based reasons. They can be classified according to a) physical impacts b) height datum inconsistencies c) processing of the satellite/radar imaging. The physical reasons can be of major importance, and they can be traced or revealed by using and combining measured and computed data of $H(\varphi,\lambda)$. The maps of these differences can be used to point out those regions where the basic accuracy of the height datum is expected to be poor:

$$\delta H(\varphi,\lambda) = (h(\varphi, \lambda)_{GPS} - N(\varphi, \lambda)_{computed}) - H(\varphi, \lambda)_{measured} \qquad (3)$$

The main cause of the difference δH comes from the fact that the computed topographic heights $H_{computed}$ at GPS positions are based on a global scale variation of the Earth's gravity field in contrast with the locally evaluated/measured heights $H_{measured}$ which depend mostly on the impact of local particularities of the terrain relief or variations of the local gravity field. Therefore differences $\delta H(\varphi,\lambda)$ are expected to express an accumulated impact from various sources, out of which the *largest part* is caused by the regional particularities having mostly physical origin (Doufexopoulou et al. 2005).

A closer look at the formula (1) shows that by using the presently available technology and knowledge of geodesy, the most sensitive factors to represent the terrain relief are a) the resolving capacity of a set of values $N(\varphi,\lambda)$ over larger regions that go beyond national borders; b) the accuracy of the set $N(\varphi,\lambda)$.

Concerning the resolving capability, its improvement in any ground pixel $\delta\varphi\delta\lambda$ is possible within the technical realm of the particular applied method of topographic surveying. The representation of values $N(\varphi,\lambda)$ within a region is used to provide a) a model for global MSL for any Earth's ellipsoidal datum that is a global reference for topographic heights H at any GPS position in the region; b) a common reference for the Earth to connect local topographic height systems in a region. The values $N(\varphi,\lambda)$ on a territory can be determined by local measurements (gravity measurements, parallel leveling and GPS measurements) or by calculations from a global model (see Eq.2.). The accuracy is determined by the instrumentation and the local gravity variations. It is reasonable to assume that locally measured or leveled topographic heights H carry more "information" related to the regional and local variations of the Earth's gravity field, which has a large impact on the *topographic relief*. In particular a set of differences $\delta H(\varphi,\lambda)$ between global and local perspectives reflects *trend variations* between a global and the regional MSL datum as well as it carries other physical impacts coming from the inner Earth's structure. In practice the topographic heights H determined by terrestrial methods are used to qualify all new forms of the topographic relief, which are produced by radar imaging and other satellite based methods. They are neglected in the present paper.

An understanding of the physical character of the terrain relief is crucial for geo-referencing but it mainly becomes useful for geographical, engineering and environmental issues. The form of terrain between *flat* and *steep* determines its suitability for settlement, farming, cost of designing transportation routes, hazards, drainage characteristics, water movement. Also the form and the quality of terrain reliefs helps determine weather patterns and in forecasting the evolution of local mass transfers. The precise knowledge of terrain reliefs on a regional and local scale is vital in aviation for low-flying routes and airport altitudes. Furthermore, flat, hilly or mountainous terrain reliefs may strongly affect the implementation of human-based installations and uses of geographical space. However, the demands for a precise representation of topographic reliefs in any of these uses are varying at a regional and a local geographical scale.

3. Sources of inconsistencies in the representation of topographic reliefs

The representation of terrain reliefs can be expressed in various forms based on individual points or on pixel values of topographic heights. These forms can be a) paper based topographic maps; b) local or regional height averages and c) methods based on satellite imaging. There are large conceptual, formal and technical differences among the used methods, which bring slightly various representation forms of Digital Terrain Models or Digital Elevation Models. They are available for several territories on the Internet (e.g. de Ferranti 2005-2012).

There are, however, two principal sources of *ambiguities* that may characterize any representation form of the terrain relief. The first is caused by combinations of various determinations of MSL over the same Earth's datum; thus, the source depends on the combined impact of available data and on the used method of computing $N(\varphi,\lambda)$. This source *spoils the consistency of regional height reference systems* by decreasing *regionally* the accuracy of the height datum at rates which may vary between some decades of cm and meters (see the investigation of Greek territory in the next part) .

The second source has its origins in technology based reasons, which have impact on the acquiring of the basic data for estimating values for $N(\varphi,\lambda)$. These concern: a) the frequencies of radar based signals; b) the height and the trajectory of the satellite *carrying the radar*; c) atmospheric conditions that affect the quality of the emitted signals; d) the physical conditions near to surface; and several other factors. All these factors are subject to particular techniques inside technology based disciplines and they affect the accuracy of satellite imaging to deduce a form of "bald" terrain. Except the type of heights, which determines the zero-level surface, all issues involve many factors to produce an *optimally combined* form of *topographic reliefs.*

The *terrain relief* has dual role in geosciences, namely a) as an irregular physical Earth's surface, viewed as *interface* between internal and external processes of an environmental Earth's system and also b) as an accurate representation of the Earth's topography which, has an *impact* on the evolution of dynamical processes. The dual role creates a need to investigate two issues for qualification of the topographic height data: a) the accuracy as third coordinate for geo-referencing and b) the accuracy and resolving capacity as a geographical variable. For simple users of "geo-referencing" a geometric height $h(\varphi,\lambda)$ results from transforming GPS X,Y,Z coordinates to an Earth's datum. A further conversion of $h(\varphi,\lambda)$ to topographic height $H(\varphi,\lambda)$ is an option of a processing algorithm with regard to offered versions of an MSL frame. Instead, the already mapped topographic heights $H(\varphi,\lambda)$ may come from many other methods based on directly or indirectly observed and acquired data including the types of image processing. When the terrain relief has the role of *geographical variable,* the questions about regional a*ccuracy* and *resolving capacity* are crucial. Obviously, it can be useful *to know in advance* those locations where are a) height datum inconsistencies and/or b) places of limited accuracy.

A relief of differences $\delta H(\varphi,\lambda)$ between the computed and the terrestrial measured topographic heights (i.e. the difference between the global gravity field model and the local raw value of $N(\varphi,\lambda)$) is expected to indicate the regional inconsistencies of the height datum and to reveal also the regions where the quality of imaging methods may be questionable due to other particular physical causes.

At present, due to the various reference coordinate frames associated with different Earth's models, and the vast sources of height data files on the web pose a real confusion to users of geo-referencing about the quality of *terrain reliefs.* Thus, parallel to the efforts to develop algorithms for 3D geo-referencing in GIS use, particular attention has to be paid to the basic quality of data sources about terrain reliefs in accuracy and resolving capacity in a region. Therefore, in spite of the apparent simplicity to convert $h(\varphi,\lambda)$ into heights $H(\varphi,\lambda)$ from GPS positioning, the terrain reliefs can be *erroneous* because of "hidden" and unresolved deviations of the height datum from the actual local ones. This situation can be frequent in many regions and there is a reasonable wish of users to have a simple method for revealing/recognizing possible locations in the geographical space where the quality of terrain reliefs should be improved.

4. Demonstration of an application

The main objective of this application is limited to use/compose differences $\delta H(\varphi,\lambda)$ at GPS and leveled positions over a region using the geoid height $N(\varphi,\lambda)$ as geographical variable/parameter. This quantity can be computed by a mathematical modeling of the Earth's gravity field (see Eq.2) and its subtraction from a GPS height $h(\varphi,\lambda)$ results to a computed topographic height $H_{computed}$ in accordance with Eq.(1).

Both topographic height sets, $H_{computed}$ and $H_{measured}$, carry information about the Earth's inner structure. However, the computed height carries global information while the measured height carries local ones. Therefore the ranked sizes of differences $\delta H(\varphi,\lambda)$ between computed and measured heights in a region can be used to point out those places where the quality of representing the terrain relief is expected poor. It is

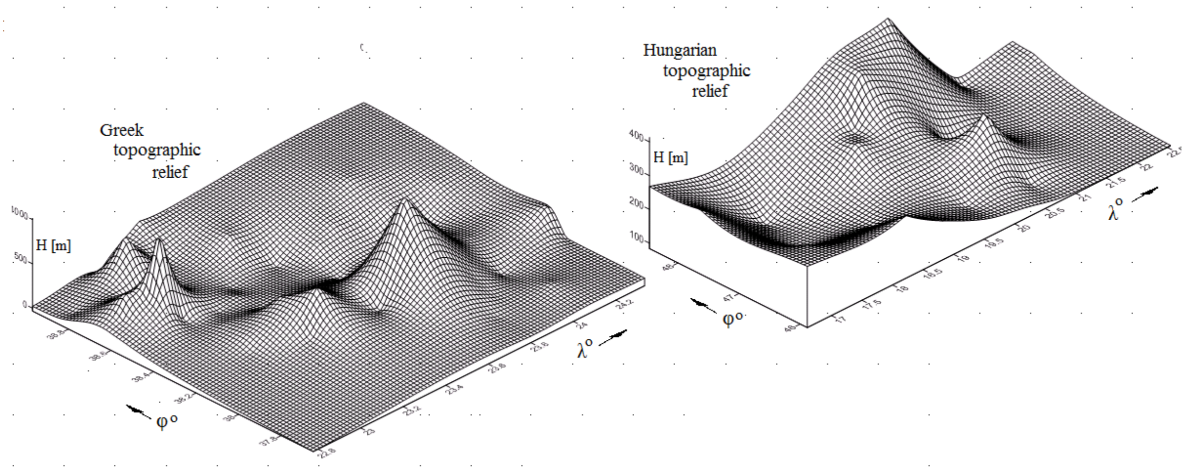

Fig.2 Topographic reliefs in Greece and Hungary

important for applications in which the slopes of topographic relief are a sensitive parameter for the evolution of dynamical phenomena within the Earth's system.

For illustration two geographical regions were selected where both, GPS and leveled heights, have been available at a number of geographical positions. The first region is located in Greece at geographical domain: $\varphi = 37.5^o$- 39^o, $\lambda = 22.5^o$- 24.5^o, H = 1-1162m; the other one is in Hungary, at geographical domain: $\varphi = 45.5^o$- 48.5^o, $\lambda = 16.5^o$- 23^o, H = 90 - 415m. There is a strong difference in the range of topographic relief between the two regions (see on Fig.2), although the range of measured geoid height : $N(\varphi,\lambda)_{measured} = h(\varphi,\lambda)_{GPS} - H(\varphi,\lambda)_{measured}$ appears similar (see on Fig.3).

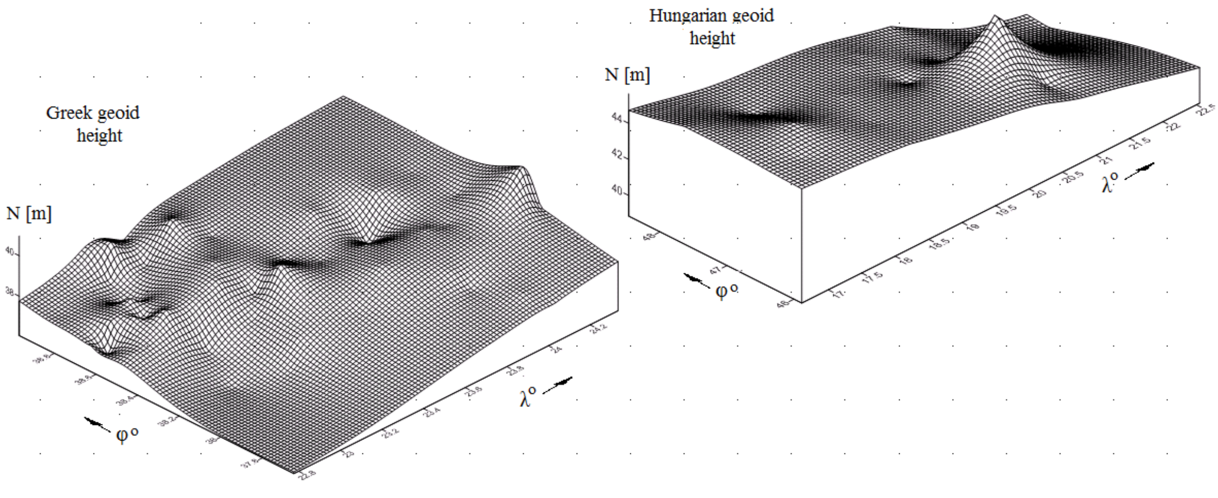

Fig.3 Measured geoid height relief in Greece and Hungary

The computed geoid heights were calculated on the base of Eq.(2) using ICGEM calculator available on the Internet (Barthelme 2009b). In the calculations WGS84 as reference system and EIGEN-GL04C as gravity model have been applied.

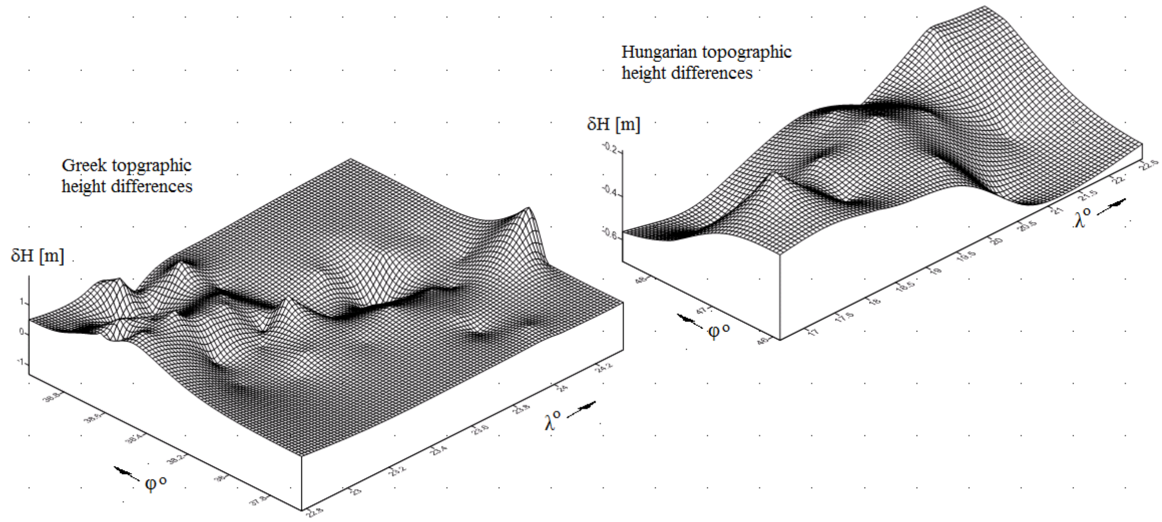

Fig.4 Topographic height differences (computed-measured) in Greece and Hungary

The differences between computed and measured heights $\delta H(\varphi,\lambda)$ (see Eq.3) for the selected places are depicted on Fig.4. The reliefs of these differences are indicating those places where computed topographic reliefs should be treated with precaution. The range of Greek relief of height difference is 3.5 m, while the Hungarian one is 0.5 m. It is apparent that the Greek relief of the height differences follows the trend of geoid height relief; what indicates *a shift in height datum*. The Hungarian relief of height difference does not reveal this similarity. Apart from some places (Northwest and Southeast part) the relief reflects random, small scale deviations.

5. Concluding remarks

Through this analysis, the principal goal of the numerical investigation aimed to illustrate existing difficulties in the unified quality to represent the *terrain relief* for geographical regions of any size when height data come from various sources. In most cases the used height data sources are not referenced to a common reference frame. Thus the produced height data files under any type of form are expected to contain two different types of ambiguities. The first type relates to the consistency of GPS height datum to measured or mapped topographic heights, which are based on past determinations of national height systems. The second type relates to the variations of resolution and of accuracy among the contributing sources of topographic heights with respect to the "real" MSL. The height of geoid, $N(\varphi,\lambda)$ is used as geographical variable, which represents a physical surface associated with a mathematical model. Thus, strong deviations $\delta H(\varphi,\lambda)$ between the local and the GPS heights represent places where height referencing of the terrain relief can be expected to be of poor quality.

The main objective of the paper is to illustrate the existence of this issue even within small areas. The application revealed that there are not negligible differences between measured and computed topographic height referencing. The strong similarity between reliefs of height difference δH and geoid height N in the Greek region indicates inconsistencies between the GPS and the national height datum.

Other larger deviations reveal strong local mass-density effects of underground features.

Such differences may occur when topographic maps are used as a main source of topographic height data to produce terrain relief files. The recognition of points or clusters of points at which large differences δH are detected can be useful to terrain relief data producers to call attention to those places where different height reference frames may introduce a systematic deviations from the GPS height datum.

Beside a previous use of topographic height differences as a geophysical signal on the local inner Earth's structure (Doufexopoulou et al. 2005), their use in qualification terrain reliefs is possible and crucial for applications which are based on the representation of terrain reliefs and its variations as a parameter of dynamical physical processes within the Earth's system (e.g. pollution, meteorology, short and long term weather forecasting, natural hazard).

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References

Barthelmes F 2009a: GFZ Scientific Technical Reports 09/02, Potsdam.

Barthelmes F 2009b: http://icgem.gfz-potsdam.de/ICGEM/

Doufexopoulou M, Massinas V, Bartha G 2005: In Geodetic Deformation Monitoring , Springer Verlag, Heidelberg

de Ferranti J 2005-2012: http://www.viewfinderpanoramas.org/dem3.html

Hofmann-Wellenhof B Moritz H 2005: Physical geodesy, Springer Verlag, Viena

Rees V G 2000: *International Journal of Remote Sensing*, 21, 1, 7-20

Strozzi L, Werner C, Wiesmann A, Wegmuller U 2012: *Geoscience and Remote Sensing Letters IEEE*, 2, 2, 277-281